Package 'StableEstim'

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Type Package

Title Estimate the Four Parameters of Stable Laws using Different Methods

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Description Estimate the four parameters of stable laws using maximum likelihood method, generalised method of moments with finite and continuum number of points, iterative Koutrouvelis regression and Kogon-McCulloch method. The asymptotic properties of the estimators (covariance matrix, confidence intervals) are also provided.

License GPL (>= 2)

RdMacros Rdpack

URL https://geobosh.github.io/StableEstim/ (doc), https://CRAN.R-project.org/package=StableEstim

BugReports https://github.com/GeoBosh/StableEstim/issues

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Contents

`StableEstim-package` *Stable law estimation functions*

Description

A collection of methods to estimate the four parameters of stable laws. The package also provides functions to compute the characteristic function and tools to run Monte Carlo simulations.

Details

The main functions of the package are briefly described below:

main function: `Estim` is the most useful function of the package. It estimates of the parameters and the asymptotic properties of the estimators.

estimation function: the methods provided so far are the maximum-likelihood (`MLParametersEstim`), the generalised method of moment with finite (`GMMParametersEstim`) or continuum (`CgmmParametersEstim`) moment conditions, the iterative Koutrouvelis regression method (`KoutParametersEstim`) and the fast Kogon-McCulloch method used for first guess estimation (`IGParametersEstim`).

characteristic function: the characteristic function (`ComplexCF`) and its Jacobian (`jacobianComplexCF`) can be computed and will return a vector (respectively a matrix) of complex numbers.

Monte Carlo simulation `Estim_Simulation` is a tool to run Monte Carlo simulations with flexible options to select the estimation method, the Monte Carlo control parameters, compute statistical summaries or save results to a file.

Note

Version 1 of this package had a somewhat restricted license since it needed package **akima** in some computations.

In version 2 of the package we implemented a 2D interpolation routine and removed the dependency on **akima**. Therefore, **StableEstim** is now under GPL license. The package is related to upcoming work by the authors where the different methods are compared using MC simulations.

Author(s)

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References

Carrasco M and Florens J (2000). "Generalization of GMM to a continuum of moment conditions." *Econometric Theory*, **16**(06), pp. 797–834.

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Carrasco M, Florens J and Renault E (2007). "Linear inverse problems in structural econometrics estimation based on spectral decomposition and regularization." *Handbook of econometrics*, **6**, pp. 5633–5751.

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Nolan JP (2012). *Stable Distributions - Models for Heavy Tailed Data*. Birkhauser, Boston. In progress, Chapter 1 online at academic2.american.edu/∼jpnolan.

Hansen LP (1982). "Large sample properties of generalized method of moments estimators." *Econometrica: Journal of the Econometric Society*, pp. 1029–1054.

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Feuerverger A and McDunnough P (1981). "On efficient inference in symmetric stable laws and processes." *Statistics and Related Topics*, **99**, pp. 109–112.

Feuerverger A and McDunnough P (1981). "On some Fourier methods for inference." *Journal of the American Statistical Association*, **76**(374), pp. 379–387.

Schmidt P (1982). "An improved version of the Quandt-Ramsey MGF estimator for mixtures of normal distributions and switching regressions." *Econometrica: Journal of the Econometric Society*, pp. 501–516.

Besbeas P and Morgan B (2008). "Improved estimation of the stable laws." *Statistics and Computing*, **18**(2), pp. 219–231.

See Also

`fBasics:::.mleStableFit`, `fBasics:::.qStableFit`

package **stabledist**

`Best_t-class` *Class* `"Best_t"`

Description

Class used to store the result of function `ComputeBest_t`.

Objects from the Class

Objects can be created by calls of the form `new("Best_t", theta, nbt, tvec, detVal, convcode, ...)`, where the user can specify some/all of the inputs or call function `ComputeBest_t`.

Slots

`theta`: Object of class `"vector"`; values of the 4 parameters.

`nbt`: Object of class `"vector"`; number of points used in the minimisation.

`tvec`: Object of class `"list"`; values of the best t-vectors.

`detVal`: Object of class `"vector"`; values of the optimal determinant found after minimisation.

`convcode`: Convergence code.

Methods

`+` `signature(e1 = "Best_t", e2 = "Best_t")`: sum objects from class `Best_t`.

initialize `signature(.Object = "Best_t")`: initialise an object from class `Best_t` as described above.

show `signature(object = "Best_t")`: print a summary of the object.

See Also

ComputeBest_t

CgmmParametersEstim *Estimate parameters of stable laws using a Cgmm method*

Description

Estimate the four parameters of stable laws using generalised method of moments based on a continuum of complex moment conditions (Cgmm) due to Carrasco and Florens. Those moments are computed by matching the characteristic function with its sample counterpart. The resulting (ill-posed) estimation problem is solved by a regularisation technique.

Usage

```
CgmmParametersEstim(x, type = c("2S", "IT", "Cue"), alphaReg = 0.01,
                    subdivisions = 50,
                    IntegrationMethod = c("Uniform", "Simpson"),
                    randomIntegrationLaw = c("unif", "norm"),
                    s_min = 0, s_max = 1,
                    theta0 = NULL,
                    IterationControl = list(),
                    pm = 0, PrintTime = FALSE,...)
```

Arguments

x	Data used to perform the estimation: a vector of length n.
type	Cgmm algorithm: "2S" is the two steps GMM proposed by Hansen(1982). "Cue" and "IT" are respectively the continuous updated and the iterative GMM proposed by Hansen, Eaton et Yaron (1996) and adapted to the continuum case.
alphaReg	Value of the regularisation parameter; numeric, default = 0.01.
subdivisions	Number of subdivisions used to compute the different integrals involved in the computation of the objective function (to minimise); numeric.
IntegrationMethod	Numerical integration method to be used to approximate the (vectorial) integrals. Users can choose between "Uniform" discretization or the "Simpson"'s rule (the 3-point Newton-Cotes quadrature rule).
randomIntegrationLaw	Probability measure associated to the Hilbert space spanned by the moment conditions. See Carrasco and Florens (2003) for more details.
s_min, s_max	Lower and Upper bounds of the interval where the moment conditions are considered; numeric.
theta0	Initial guess for the 4 parameters values: vector of length 4.

`IterationControl`
: Only used with `type = "IT"` or `type = "Cue"` to control the iterations, see Details.

`pm`
: Parametrisation, an integer (0 or 1); default: `pm = 0` (Nolan's 'S0' parametrisation).

`PrintTime`
: Logical flag; if set to TRUE, the estimation duration is printed out to the screen in a readable format (h/min/sec).

`...`
: Other arguments to be passed to the optimisation function and/or to the integration function.

Details

The moment conditions The moment conditions are given by:

$$g_t(X, \theta) = g(t, X; \theta) = e^{itX} - \phi_\theta(t)$$

If one has a sample $x_1, \ldots, x_n$ of i.i.d realisations of the same random variable X, then:

$$\hat{g}_n(t, \theta) = \frac{1}{n} \sum_{i=1}^{n} g(t, x_i; \theta) = \phi_n(t) - \phi_\theta(t),$$

where $\phi_n(t)$ is the eCF associated with the sample $x_1, \ldots, x_n$, defined by $\phi_n(t) = \frac{1}{n} \sum_{j=1}^{n} e^{itX_j}$.
Objective function

Following Carrasco et al. (2007), Proposition 3.4, the objective function to minimise is given by:

$$obj(\theta) = \overline{\underline{v}'}(\theta)[\alpha_{Reg}\mathcal{I}_n + C^2]^{-1}\underline{v}(\theta)$$

where:

$\underline{v} = [v_1, \ldots, v_n]'$; $v_i(\theta) = \int_I \overline{g_i}(t; \hat{\theta}_n^1)\hat{g}(t; \theta)\pi(t)dt$.

I_n is the identity matrix of size n.

C is a $n \times n$ matrix with (i, j)th element given by $c_{ij} = \frac{1}{n-4} \int_I \overline{g_i}(t; \hat{\theta}_n^1) g_j(t; \hat{\theta}_n^1)\pi(t)dt$.

To compute C and $v_i()$ we will use the function `IntegrateRandomVectorsProduct`. **The IterationControl** If `type = "IT"` or `type = "Cue"`, the user can control each iteration using argument `IterationControl`, which should be a `list` which contains the following elements:

`NbIter`**:** maximum number of iterations. The loop stops when `NBIter` is reached; default = 10.

`PrintIterlogical`**:** if set to TRUE the values of the current parameter estimates are printed to the screen at each iteration; default = TRUE.

`RelativeErrMax`**:** the loop stops if the relative error between two consecutive estimation steps is smaller then `RelativeErrMax`; default = 1e-3.

Value

a list with the following elements:

`Estim`
: output of the optimisation function,

`duration`
: estimation duration in numerical format,

`method`
: `character` describing the method used.

Note

nlminb as used to minimise the Cgmm objective function.

References

Carrasco M, Florens J (2000). "Generalization of GMM to a continuum of moment conditions." *Econometric Theory*, **16**(06), 797–834.

Carrasco M, Florens J (2002). "Efficient GMM estimation using the empirical characteristic function." *IDEI Working Paper*, **140**.

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Carrasco M, Kotchoni R (2010). "Efficient estimation using the characteristic function." Mimeo. University of Montreal.

See Also

Estim, GMMParametersEstim, IntegrateRandomVectorsProduct

Examples

```
## general inputs
theta <- c(1.45, 0.55, 1, 0)
pm <- 0
set.seed(2345)
x <- rstable(50, theta[1], theta[2], theta[3], theta[4], pm)

## GMM specific params
alphaReg <- 0.01
subdivisions <- 20
randomIntegrationLaw <- "unif"
IntegrationMethod <- "Uniform"

## Estimation
twoS <- CgmmParametersEstim(x = x, type = "2S", alphaReg = alphaReg,
                            subdivisions = subdivisions,
                            IntegrationMethod = IntegrationMethod,
                            randomIntegrationLaw = randomIntegrationLaw,
                            s_min = 0, s_max = 1, theta0 = NULL,
                            pm = pm, PrintTime = TRUE)
twoS
```

ComplexCF *Compute the characteristic function of stable laws*

Description

Theoretical characteristic function (CF) of stable laws under parametrisation 'S0' or 'S1'. See Nolan (2013) for more details.

Usage

```
ComplexCF(t, theta, pm = 0)
```

Arguments

t	vector of (real) numbers where the CF is evaluated; numeric.
theta	vector of parameters of the stable law; vector of length 4.
pm	parametrisation, an integer (0 or 1); default: pm = 0 (Nolan's 'S0' parametrisation).

Details

For more details about the different parametrisation of the CF, see *Nolan(2012)*.

Value

vector of complex numbers with dimension length(t).

References

Nolan JP (2012). *Stable Distributions - Models for Heavy Tailed Data*. Birkhauser, Boston. In progress, Chapter 1 online at academic2.american.edu/∼jpnolan.

See Also

jacobianComplexCF

Examples

```
## define the parameters
nt <- 10
t <- seq(0.1, 3, length.out = nt)
theta <- c(1.5, 0.5, 1, 0)
pm <- 0

## Compute the characteristic function
CF <- ComplexCF(t = t, theta = theta, pm = pm)
CF
```

`ComputeBest_t` *Monte Carlo simulation to investigate the optimal number of points to use in the moment conditions*

Description

Runs Monte Carlo simulation for different values of α and β and computes a specified number of t-points that minimises the determinant of the asymptotic covariance matrix.

Usage

```
ComputeBest_t(AlphaBetaMatrix = abMat, nb_ts = seq(10, 100, 10),
              alphaReg = 0.001, FastOptim = TRUE, ...)
```

Arguments

`AlphaBetaMatrix`	values of the parameter α and β from which we simulate the data. By default, the values of γ and δ are set to 1 and 0, respectively; a $2 \times n$ matrix.
`nb_ts`	vector of numbers of t-points to use for the minimisation; default = `seq(10, 100, 10)`.
`alphaReg`	value of the regularisation parameter; numeric, default = 0.001.
`FastOptim`	Logical flag; if set to TRUE, `optim` with "Nelder-Mead" method is used (fast but not accurate). Otherwise, `nlminb` is used (more accurate but slower).
`...`	Other arguments to pass to the optimisation function.

Value

a `list` containing slots from class `Best_t-class` corresponding to one value of the parameters α and β.

See Also

`ComputeBest_tau`, `Best_t-class`

`ComputeBest_tau` *Run Monte Carlo simulation to investigate the optimal τ*

Description

Runs Monte Carlo simulation to investigate the optimal number of points to use when one of the reduced spacing schemes is considered.

Usage

```
ComputeBest_tau(AlphaBetaMatrix = abMat, nb_ts = seq(10, 100, 10),
                tScheme = c("uniformOpt", "ArithOpt"),
                Constrained = TRUE, alphaReg = 0.001, ...)
```

Arguments

`AlphaBetaMatrix`	values of the parameter α and β from which we simulate the data. By default, the values of γ and δ are set to 1 and 0, respectively; a $2 \times n$ matrix.
`nb_ts`	vector of number of t-points to use for the minimisation; default = `seq(10,100,10)`.
`tScheme`	scheme used to select the points where the moment conditions are evaluated, one of `"uniformOpt"` (uniform optimal placement) and `"ArithOpt"` (arithmetic optimal placement). See function `GMMParametersEstim`.
`Constrained`	logical flag: if set to True, lower and upper bands will be computed as discussed for function `GMMParametersEstim`.
`alphaReg`	value of the regularisation parameter; numeric, default = 0.001.
`...`	Other arguments to pass to the optimisation function.

Value

a `list` containing slots from class `Best_t-class` corresponding to one value of the parameters α and β.

See Also

`ComputeBest_t`, `Best_t-class`

`ComputeDuration` *Duration*

Description

Compute the duration between 2 time points.

Usage

```
ComputeDuration(t_init, t_final, OneNumber = FALSE)
```

Arguments

`t_init`	Starting time; numeric.
`t_final`	Final time; numeric.
`OneNumber`	Logical flag; if set to TRUE, the duration in seconds will be returned. Otherwise, a vector of length 3 will be computed representing the time in h/min/sec.

Value

a numeric of length 1 or 3 depending on the value of OneNumber flag.

See Also

PrintDuration, PrintEstimatedRemainingTime.

Examples

```
ti <- getTime_()
for (i in 1:100) x <- i*22.1
tf <- getTime_()
ComputeDuration(ti,tf)
```

ComputeFirstRootRealeCF

First root of the empirical characteristic function

Description

Computes the first root of the real part of the empirical characteristic function.

Usage

```
ComputeFirstRootRealeCF(x, ..., tol = 0.001, maxIter = 100,
                        lowerBand = 1e-04, upperBand = 30)
```

Arguments

x	data used to perform the estimation: vector of length n.
...	other arguments to pass to the optimisation function.
tol	tolerance to accept the solution; default = 1e-3.
maxIter	maximum number of iteration in the Welsh algorithm; default = 100.
lowerBand	lower band of the domain where the graphical seach is performed; default = 1e-4.
upperBand	Lower band of the domain where the graphical seach is performed; default = 30.

Details

The Welsh algorithm is first applied. If it fails to provide a satisfactory value (< tol), a graphical/ numerical approach is used. We first plot the real part of the eCF vs t in order to determine the first zero directly and use it as the initial guess of a numerical minimisation routine.

Value

numeric: first zero of the real part of the eCF.

References

Welsh AH (1986). "Implementing empirical characteristic function procedures." *Statistics & probability letters*, **4**(2), 65–67.

See Also

ComplexCF

Examples

```
set.seed(345)
x <- rstable(500, 1.5, 0.5)
ComputeFirstRootRealeCF(x)
```

ComputeStatObjectFromFiles

Parse an output file to create a summary object (`list`*)*

Description

Parses the file saved by Estim_Simulation and re-creates a summary list identical to the one produced by Estim_Simulation when StatSummary is set to TRUE.

Usage

```
ComputeStatObjectFromFiles(files, sep_ = ",",
                           FctsToApply = StatFcts,
                           headers_=TRUE,readSizeFrom=1,
                           CheckMat=TRUE,
                           tolFailCheck=tolFailure,
                           MCparam=1000,...)
```

Arguments

files	character vector containing the files name to be parsed. See Details.
sep_	field separator character to be used in function read.csv() and write.table(). Values on each line of the file are separated by this character. It can also be a character vector (same length as files) if different separators are used for each file; default: ",".
FctsToApply	functions used to produce the statistical summary. See Estim_Simulation; character vector.
headers_	boolean vector of length 1 or same length as files to indicate for each file if the header argument is to be considered or not. To be passed to function read.csv().
readSizeFrom	index of the file from which the sample sizes are determined; default 1 (from first file in files).

CheckMat	logical flag: if set to TRUE, an estimation is declared failed if the squared error of the estimation is larger than tolFailCheck; default TRUE.
tolFailCheck	tolerance on the squared error of the estimation to be declared failed; default = 1.5.
MCparam	number of Monte Carlo simulation for each couple of parameter, default = 1000; integer.
...	other arguments to be passed to the estimation function. See Estim_Simulation.

Details

The same sample sizes are assumed for all the files and we also assume a different set of parameters (alpha,beta) within each file (one and one only).

This function is particularly useful when simulations are run in parallel on different computers/CPUs and the output files are collected afterwards. This function is also used to create the Latex summary table: see TexSummary.

Some examples are provided in the example folder.

Value

a list of length 4 containing a summary matrix object associated to each parameter.

See Also

Estim_Simulation

ConcatFiles *Concatenates output files.*

Description

Creates a unique file by concatenating several output files associated to one set of parameters.

Usage

```
ConcatFiles(files, sep_ = ",", outfile, headers_ = TRUE,
            DeleteIfExists=TRUE)
```

Arguments

files	character Vector containing the files name to be concatenated. See details.
sep_	Field separator character to be used in function read.csv() and write.table(). Values on each line of the file are separated by this character; It can also be a vector character (same length as files) if different separators are useed for each file; default: ","
outfile	Name of the output file; character

headers_	Vector of boolean of length 1 or same length as files to indicate for each file if the header argument is to be considered or not. To be passed to function read.csv().
DeleteIfExists	if outfile exists, it will be deleted and recreated (over-written).

Details

The files to be concatenated should be related to the same set of parameters alpha and beta. The function stops if one of the file contains 2 (or more) different set of parameters (the function compares the values of columns 1 and 2 row by row) or if the set of parameters within one file is different from the one from other files.

Value

Returns an output file outfile saved in the working directory.

See Also

Estim_Simulation

Estim *Estimate parameters of stable laws*

Description

Estimates the four parameters of stable distributions using one of the methods implemented in **StableEstim**. This is the main user-level function but the individul methods are available also as separate functions.

Usage

```
Estim(EstimMethod = c("ML", "GMM", "Cgmm","Kout"), data, theta0 = NULL,
      ComputeCov = FALSE, HandleError = TRUE, ...)
```

Arguments

EstimMethod	Estimation method to be used, one of "ML" (maximum likelihood, default), "GMM" (generalised method of moment with finite moment conditions), "Cgmm" (GMM with continuum moment conditions), and "Kout" (Koutrouvelis regression method).
data	Data used to perform the estimation, a numeric vector.
theta0	Initial values for the 4 parameters. If NULL (default), initial values are computed using the fast Kogon-McCulloch method, see IGParametersEstim; vector of length 4.
ComputeCov	Logical flag: if TRUE, the asymptotic covariance matrix (4x4) is computed (except for the Koutrouvelis method).

HandleError	Logical flag: if TRUE and if an error occurs during the estimation procedure, the computation will carry on and NA will be returned. Useful for Monte Carlo simulations, see Estim_Simulation.
...	Other arguments to be passed to the estimation function, such as the asymptotic confidence level, see Details.

Details

Estim is the main estimation function in package **StableEstim**.

This function should be used in priority for estimation purpose as it provides more information about the estimator. However, user needs to pass the appropriate parameters to the selected method in .... See the documentation of the selected method.

Asymptotic Confidence Intervals: The *normal* asymptotic confidence intervals (CI) are computed. The user can set the *level* of confidence by inputing the level argument (in the "\dots"); default level=0.95. The theoretical justification for asymptotic normal CI can be found in the references for the individual methods. Note the CI's are not computed for the Koutrouvelis regression method.

Value

an object of class Estim, see Estim-class for more details

See Also

CgmmParametersEstim, GMMParametersEstim, MLParametersEstim, KoutParametersEstim for the individual estimation methods;

IGParametersEstim for fast computation of initial values.

Examples

```
## general inputs
theta <- c(1.45, 0.55, 1, 0)
pm <- 0
set.seed(2345)
x <- rstable(200, theta[1], theta[2], theta[3], theta[4], pm)

objKout <- Estim(EstimMethod = "Kout", data = x, pm = pm,
                     ComputeCov = FALSE, HandleError = FALSE,
                     spacing = "Kout")
```

Estim-class *Class* "Estim"

Description

Class for storing the results of estimating parameters of stable laws, output of function Estim().

Objects from the Class

Objects can be created by calls of the form `new("Estim", par, ...)`. Users can provide some (or all) of the inputs stated below to create an object from this class or call function `Estim` with appropriate arguments.

Slots

`par`: `numeric(4)`, values of the 4 estimated parameters.

`par0`: `numeric(4)`, initial values for the 4 parameters.

`vcov`: object of class `"matrix"` (4 x 4), representing the covariance matrix of the estimated parameters.

`confint`: object of class `"matrix"` (4 x 4), representing the confidence interval computed at a specific level (attribute of the object).

`data`: `numeric()`, the data used to compute the estimates.

`sampleSize`: `numeric(1)`, length of the data.

`others`: `list()`, further information about the estimation method.

`duration`: `numeric(1)`, duration in seconds.

`failure`: `numeric(1)`, represents the status of the procedure: 0 failure or 1 success.

`method`: Object of class `"character"`, description of the parameters used in the estimation.

Methods

initialize `signature(.Object = "Estim")`: creates an object of this class using the inputs described above.

show `signature(object = "Estim")`: summarised print of the object.

See Also

`Estim`

`Estim_Simulation` *Monte Carlo simulation*

Description

Runs Monte Carlo simulation for a selected estimation method. The function can save a file and produce a statistical summary.

Usage

```
Estim_Simulation(AlphaBetaMatrix = abMat, SampleSizes = c(200, 1600),
                 MCparam = 100, Estimfct = c("ML", "GMM", "Cgmm","Kout"),
                 HandleError = TRUE, FctsToApply = StatFcts,
                 saveOutput = TRUE, StatSummary = FALSE,
                 CheckMat = TRUE, tolFailCheck = tolFailure,
                 SeedOptions=NULL, ...)
```

Arguments

`AlphaBetaMatrix`	values of the parameter α and β from which we simulate the data. By default, the values of γ and δ are set to 1 and 0, respectively; a $2 \times n$ matrix.
`SampleSizes`	sample sizes to be used to simulate the data. By default, we use `200` (small sample size) and `1600` (large sample size); vector of integers.
`MCparam`	Number of Monte Carlo simulation for each couple of parameter, default = 100; an integer number.
`Estimfct`	the estimation function to be used, one of `"ML"`, `"GMM"`, `"Cgmm"` or `"Kout"`.
`HandleError`	logical flag: if set to TRUE, the simulation doesn't stop when an error in the estimation function is encountered. A vector of (size 4) `NA` is saved and the the simulation carries on. See details.
`FctsToApply`	functions used to produce the statistical summary. See details; a character vector.
`saveOutput`	logical flag: if set to TRUE, a csv file (for each couple of parameters α and β) with the the estimation information is saved in the current directory. See Details.
`StatSummary`	logical flag: if set to TRUE, a statistical summary (using `FctsToApply`) is returned. See Details.
`CheckMat`	logical flag: if set to TRUE, an estimation is declared failed if the squared error of the estimation is larger than `tolFailCheck`; default = TRUE.
`tolFailCheck`	tolerance on the squared error of the estimation to be declared failed; default = 1.5.
`SeedOptions`	list to control the seed generation. See Details.
`...`	other arguments to be passed to the estimation function.

Details

Error Handling It is advisable to set it to TRUE when the user is planning to launch long simulations as it will prevent the procedure from stopping if an error occurs for one sample data. The estimation function will produce a vector of `NA` as estimated parameters related to this (error generating) sample data and move on to the next Monte Carlo step. **Statistical summary** The function is able to produce a statistical summary of the Monte Carlo simulation for each parameter (slices of the list). Each slice is a matrix where the rows represents the true values of the parameters and the columns the statistical information. In all cases, the following quantities are computed:

`sample size`**:** the sample size used to produce the simulated data.

`alphaT`**,** `betaT`**:** the true values of the parameters.

`failure`**:** the number of times the procedure failed to produce relevant estimation.

`time`**:** the average running time in seconds of the estimation procedure

Besides, the (vector of `character`) `FctsToApply` controls the other quantities to be computed by providing the name of the function object to be applied to the vector of estimated parameters. The signature of the function should be of the form `fctName = function(p,...){...}`, where `p` is the vector (`length(p) = MCparam`) of parameter estimates and `...` is the extra arguments to be passed the function.

By default, the functions from StatFcts will be applied but the user can pass his own functions by providing their names in argument FctsToApply and their definitions in the global environment.

Note that if CheckMat is set to TRUE, the estimation is considered failed if the squared error (of the first 2 parameters alpha and beta) is larger than tolFailCheck.

Output file

Setting saveOutput to TRUE will have the side effect of saving a csv file in the working directory. This file will have MCparam * length(SampleSizes) lines and its columns will be:

alphaT, betaT: the true values of the parameters.

data size: the sample size used to generate the simulated data.

seed: the seed value used to generate the simulated data.

alphaE, betaE, gammaE, deltaE: the estimates of the 4 parameters.

failure: binary: 0 for success, 1 for failure.

time: estimation running time in seconds.

The file name is informative to let the user identify the values of the true parameters, the MC parameters, as well as the options selected for the estimation method.

The csv file is updated after each MC estimation, which is useful when the simulation stops before it finishes. Besides, using the check-pointing mechanism explained below, the simulation can re-start from where it stopped.

Check-pointing. Checkpointing is the act of saving enough program state and results so far calculated that a computation can be stopped and restarted. The way we did it here is to save a text file with some useful information about the state of the estimation. This text file is updated after each MC iteration and read at the beginning of function Estim_Simulation to allow the simulation to re-start from where it stopped. This file is deleted at the end of the simulation procedure.

SeedOptions. Users who do not want to control the seed generation can ignore this argument (its default value is NULL). This argument can be more useful when one wants to cut the simulation (even for one parameter value) into pieces. In that case, the user can control which part of the seed vector to use.

MCtot: total values of MC simulations in the entire process.

seedStart: starting index in the seed vector. The vector extracted will be of size MCparam.

Value

If StatSummary is set to TRUE, a list with 4 components (corresponding to the 4 parameters) is returned. Each component is a matrix. If SaveOutput is set to TRUE, only a csv file is saved and nothing is returned (if StatSummary is FALSE). If both are FALSE, the function stops.

See Also

Estim, CgmmParametersEstim, GMMParametersEstim, MLParametersEstim

expect_almost_equal *Test approximate equality*

Description

Tests the approximate equality of 2 objects. Useful for running tests.

Usage

```
expect_almost_equal(x, y, tolExpect = 0.001)
```

Arguments

x	first object.
y	second object.
tolExpect	tolerance, default is 0.001.

Details

This function works with the `expect_that` function from package `testhat` to test equality between 2 objects with a given tolerance. It is used particularly for testing functions output. See the CF examples in the Examples folder.

See Also

`expect_that`, `testthat`

Examples

```
x <- 1.1
y <- 1.5
expect_almost_equal(x, y, 1)      # passes
## expect_almost_equal(x, y, 0.3) # fails
```

get.abMat *Default set of parameters to pass to* Estim_Simulation

Description

Default set of parameters to pass to Estim_Simulation, inspired by the one used by Koutrevelis (1980) in his simulation procedure.

Usage

```
get.abMat()
```

Value

a 2-columns matrix containing a wide range of parameters α and β covering the entire parameters space.

`get.StatFcts` *Default functions used to produce the statistical summary*

Description

Default functions used to produce the statistical summary in the Monte Carlo simulations.

Usage

```
get.StatFcts()
```

Value

The functions computed are:

Mean `.mean <- function(p,...) mean(p)`

Min `.min <- function(p,...) min(p)`

Max `.max <- function(p,...) max(p)`

Sn `.Sn <- function(p,n,...) sqrt(n)*sd(p)`

MSE `.MSE <- function(p,paramT,...) (1/length(p))*sum((p-paramT)^2)`

Std error `.st.err <- function(p,...) sd(p)/sqrt(length(p))`

Users can define their own summaries by defining functions with similar signatures and passing a `character` vector containing the functions' names to `Estim_Simulation`.

`getTime_` *Read time*

Description

Reads the time when the function is called.

Usage

```
getTime_()
```

Value

a `numeric`.

See Also

PrintDuration, PrintEstimatedRemainingTime, ComputeDuration

Examples

```
ti <- getTime_()
```

GMMParametersEstim *Estimate parameters of stable laws using a GMM method*

Description

Estimate parameters of stable laws using generalised method of moments (GMM) with finite number of moment conditions. It uses a regularisation technique to make the method more robust (when the number of moment condition is large) and allows different schemes to select where the moment conditions are computed.

Usage

```
GMMParametersEstim(x, algo = c("2SGMM", "ITGMM", "CueGMM"),
                   alphaReg = 0.01,
                   regularization = c("Tikhonov", "LF", "cut-off"),
                   WeightingMatrix = c("OptAsym", "DataVar", "Id"),
                   t_scheme = c("equally", "NonOptAr", "uniformOpt",
                                "ArithOpt", "VarOpt", "free"),
                   theta0 = NULL,
                   IterationControl = list(),
                   pm = 0, PrintTime = FALSE, ...)
```

Arguments

x	data used to perform the estimation: vector of length n.
algo	GMM algorithm: "2SGMM" is the two step GMM proposed by Hansen(1982). "CueGMM" and "ITGMM" are respectively the continuous updated and the iterative GMM proposed by Hansen, Eaton et Yaron (1996) and adapted to the continuum case.
alphaReg	value of the regularisation parameter; numeric, default = 0.01.
regularization	regularization scheme to be used, one of "Tikhonov" (Tikhonov), "LF" (Landweber-Fridmann) and "cut-off" (spectral cut-off). See RegularisedSol.
WeightingMatrix	type of weighting matrix used to compute the objective function, one of "OptAsym" (the optimal asymptotic), "DataVar" (the data driven) and "Id" (the identity matrix). See Details.

`t_scheme` scheme used to select the points where the moment conditions are evaluated, one of `"equally"` (equally placed), `"NonOptAr"` (non optimal arithmetic placement), `"uniformOpt"` (uniform optimal placement), `"ArithOpt"` (arithmetic optimal placement), `"Var Opt"` (optimal variance placement) and `"free"` (users need to pass their own set of points in . . .). See Details.

`theta0` initial guess for the 4 parameters values: if `NULL`, the Kogon-McCulloch method is called, see `IGParametersEstim`; vector of length 4.

`IterationControl` only used if `type = "IT"` or `type = "Cue"` to control the iterations. See Details.

`pm` parametrisation, an integer (0 or 1); default: `pm = 0` (Nolan's 'S0' parametrisation).

`PrintTime` logical flag; if set to TRUE, the estimation duration is printed out to the screen in a readable format (h/min/sec).

`...` other arguments to pass to the regularisation function, the optimisation function or the selection scheme (including the function that finds the first zero of the eCF). See Details.

Details

The moment conditions

The moment conditions are given by:

$$g_t(X, \theta) = g(t, X; \theta) = e^{itX} - \phi_\theta(t)$$

If one has a sample $x_1, \ldots, x_n$ of i.i.d realisations of the same random variable X, then:

$$\hat{g}_n(t, \theta) = \frac{1}{n} \sum_{i=1}^{n} g(t, x_i; \theta) = \phi_n(t) - \phi_\theta(t),$$

where $\phi_n(t)$ is the eCF associated to the sample $x_1, \ldots, x_n$, and defined by $\phi_n(t) = \frac{1}{n} \sum_{j=1}^{n} e^{itX_j}$.

Objective function

$$obj\theta =< K^{-1/2}\hat{g}_n(.;\theta), K^{-1/2}\hat{g}_n(.;\theta) >,$$

where $K^{-1}f$ denotes the solution φ (when it exists) of the equation $K\varphi = f$ and $K^{-1/2} = (K^{-1})^{1/2}$. The optimal choice of the Weighting operator K (a matrix in the GMM case) and its estimation are discussed in Hansen (1982).

Weighting operator (Matrix)

`OptAsym`**:** the optimal asymptotic choice as described by Hansen. The expression of the components of this matrix could be found for example in Feuerverger and McDunnough (1981b).

`DataVar`**:** the covariance matrix of the data provided.

`Id`**:** the identity matrix.

the t-scheme

One of the most important features of this method is that it allows the user to choose how to place the points where the moment conditions are evaluated. The general rule is that users can provide their

own set of points (option "free") or choose one of the other schemes. In the latter case they *need to specify the number of points* nb_t in argument "\dots" and eventually the lower and upper limit (by setting Constrained to FALSE and providing min_t and max_t) in the non-optimised case. If one of the optimised cases is selected, setting Constrained to FALSE will not constrain the choice of τ, see below. We mean by optimised set of point, the set that minimises the (determinant) of the asymptotic covariance matrix as suggested by Schmidt (1982) and Besbeas and Morgan (2008).

6 options have been implemented:

"equally": equally placed points in [min_t,max_t]. When provided, user's min_t and max_t will be used (when Coinstrained = FALSE). Otherwise, eps and An will be used instead (where An is the first zero of the eCF).

"NonOptAr": non optimal arithmetic placement: $t_j = \frac{j(j+1)}{nbt(nbt+1)}(max - eps); j = 1, \ldots, nbt$, where max is the upper band of the set of points selected as discussed before.

"uniformOpt": uniform optimal placement: $t_j = j\tau, j = 1, \ldots, nbt$

"ArithOpt": arithmetic optimal placement: $t_j = j(j+1)\tau, j = 1, \ldots nbt$

"Var Opt": optimal variance placement as explained above.

"free": user needs to pass his own set of points in "\dots".

For the "ArithOpt" and "uniformOpt" schemes, the function to minimise is seen as a function of the real parameter τ instead of doing a vectorial optimisition as in the "Var Opt" case. In the latter case, one can choose between a fast (but less accurate) optimisation routine or a slow (but more accurate) one by setting the FastOptim flag to the desired value.

The IterationControl

If type = "IT" or type = "Cue" the user can control each iteration by setting up the list IterationControl which contains the following elements:

NbIter: maximum number of iteration. The loop stops when NBIter is reached; default = 10.

PrintIterlogical: if set to TRUE, the value of the current parameter estimation is printed to the screen at each iteration; default = TRUE.

RelativeErrMax: the loop stops if the relative error between two consecutive estimation steps is smaller than RelativeErrMax; default = 1e-3.

Value

a list with the following elements:

Estim	output of the optimisation function.
duration	estimation duration in a numerical format.
method	character describing the method used.
tEstim	final set of points selected for the estimation. Only relevant when one of the optimisation scheme is selected.

Note

nlminb was used for the minimisation of the GMM objective funcion and to compute tau in the "uniformOpt" and "ArithOpt" schemes. In the "Var Opt" scheme, optim was preferred. All those routines have been selected after running different tests using the summary table produced by package **optimx** for comparing the performance of different optimisation methods.

References

Hansen LP (1982). "Large sample properties of generalized method of moments estimators." *Econometrica: Journal of the Econometric Society*, pp. 1029–1054.

Hansen LP, Heaton J and Yaron A (1996). "Finite-sample properties of some alternative GMM estimators." *Journal of Business & Economic Statistics*, **14**(3), pp. 262–280.

Feuerverger A and McDunnough P (1981). "On efficient inference in symmetric stable laws and processes." *Statistics and Related Topics*, **99**, pp. 109–112.

Feuerverger A and McDunnough P (1981). "On some Fourier methods for inference." *Journal of the American Statistical Association*, **76**(374), pp. 379–387.

Schmidt P (1982). "An improved version of the Quandt-Ramsey MGF estimator for mixtures of normal distributions and switching regressions." *Econometrica: Journal of the Econometric Society*, pp. 501–516.

Besbeas P and Morgan B (2008). "Improved estimation of the stable laws." *Statistics and Computing*, **18**(2), pp. 219–231.

See Also

Estim, CgmmParametersEstim

Examples

```
## General data
theta <- c(1.5, 0.5, 1, 0)
pm <- 0
set.seed(345);
x <- rstable(100, theta[1], theta[2], theta[3], theta[4], pm)
##---------------- 2S free ----------------
## method specific arguments
regularization <- "cut-off"
WeightingMatrix <- "OptAsym"
alphaReg <- 0.005
t_seq <- seq(0.1, 2, length.out = 12)

## If you are just interested by the value
## of the 4 estimated parameters
t_scheme = "free"

algo = "2SGMM"
suppressWarnings(GMMParametersEstim(
    x = x, algo = algo, alphaReg = alphaReg,
    regularization = regularization,
    WeightingMatrix = WeightingMatrix,
    t_scheme = t_scheme,
    pm = pm, PrintTime = TRUE, t_free = t_seq))

## algo = "CueGMM"
GMMParametersEstim(
    x = x, algo = "CueGMM", alphaReg = alphaReg,
    regularization = regularization,
```

```
WeightingMatrix = WeightingMatrix,
t_scheme = t_scheme,
pm = pm, PrintTime = TRUE, t_free = t_seq)
```

`IGParametersEstim` *Estimate parameters of stable laws by Kogon and McCulloch methods*

Description

Kogon regression method is used together with the McCulloch quantile method to provide initial estimates of parameters of stable distributions.

Usage

```
IGParametersEstim(x, pm = 0, ...)
```

Arguments

`x`	data used to perform the estimation: vector of length n.
`pm`	parametrisation, an integer (0 or 1); default: `pm = 0` (Nolan's 'S0' parametrisation).
`...`	other arguments. Currently not used.

Details

The parameters γ and δ are estimated using the McCulloch(1986) quantile method from **fBasics**. The data is rescaled using those estimates and used to perform the Kogon regression method to estimate α and β.

Value

a vector of length 4 containing the estimates of the 4 parameters.

References

Kogon SM and Williams DB (1998). "Characteristic function based estimation of stable distribution parameters." *A practical guide to heavy tailed data*, pp. 311–335. McCulloch JH (1986). "Simple consistent estimators of stable distribution parameters." *Communications in Statistics-Simulation and Computation*, **15**(4), pp. 1109–1136.

See Also

`Estim`, `McCullochParametersEstim`

Examples

```
x <- rstable(200, 1.2, 0.5, 1, 0, pm = 0)
IGParametersEstim(x, pm = 0)
```

`IntegrateRandomVectorsProduct` *Integral outer product of random vectors*

Description

Computes the integral outer product of two possibly complex random vectors.

Usage

```
IntegrateRandomVectorsProduct(f_fct, X, g_fct, Y, s_min, s_max,
                              subdivisions = 50,
                              method = c("Uniform", "Simpson"),
                              randomIntegrationLaw = c("norm","unif"),
                              ...)
```

Arguments

`f_fct`	function object with signature `f_fct=function(s,X)` and returns a matrix $ns \times nx$ where nx=`length(X)` and ns=`length(s)`; `s` is the points where the integrand is evaluated.
`X`	random vector where the function `f_fct` is evaluated. See Details.
`g_fct`	function object with signature `g_fct=function(s,Y)` and returns a matrix $ns \times ny$ where ny=`length(Y)` and ns=`length(s)`; `s` is the points where the integrand is evaluated.
`Y`	random vector where the function `g_fct` is evaluated. See Details.
`s_min`, `s_max`	limits of integration. Should be finite.
`subdivisions`	maximum number of subintervals.
`method`	numerical integration rule, one of `"uniform"` (fast) or `"Simpson"` (more accurate quadratic rule).
`randomIntegrationLaw`	Random law pi(s) to be applied to the Random product vector, see Details. Choices are `"unif"` (uniform) and `"norm"` (normal distribution).
`...`	other arguments to pass to random integration law. Mainly, the mean (`mu`) and standard deviation (`sd`) of the normal law.

Details

The function computes the $nx \times ny$ matrix $C = \int_{s_{min}}^{s_{max}} f_s(X) g_s(Y) \pi(s) ds$, such as the one used in the objective function of the Cgmm method. This is essentially an outer product with with multiplication replaced by integration.

There is no function in R to compute vectorial integration and computing C element by element using `integrate` may be very slow when `length(X)` (or `length(y)`) is large.

The function allows complex vectors as its integrands.

Value

an $nx \times ny$ matrix C with elements:

$$c_{ij} = \int_{s_{min}}^{s_{max}} f_s(X_i) g_s(Y_j) \pi(s) ds.$$

Examples

```
## Define the integrand
f_fct <- function(s, x) {
    sapply(X = x, FUN = sampleComplexCFMoment, t = s, theta = theta)
}
f_bar_fct <- function(s, x) Conj(f_fct(s, x))

## Function specific arguments
theta <- c(1.5, 0.5, 1, 0)
set.seed(345)
X <- rstable(3, 1.5, 0.5, 1, 0)
s_min <- 0;
s_max <- 2
numberIntegrationPoints <- 10
randomIntegrationLaw <- "norm"

Estim_Simpson <-
    IntegrateRandomVectorsProduct(f_fct, X, f_bar_fct, X, s_min, s_max,
                                  numberIntegrationPoints,
                                  "Simpson", randomIntegrationLaw)
Estim_Simpson
```

---

jacobianComplexCF *Jacobian of the characteristic function of stable laws*

Description

Numeric jacobian of the characteristic function (CF) as a function of the parameter θ evaluated at a specific (vector) point `t` and a given value θ.

Usage

```
jacobianComplexCF(t, theta, pm = 0)
```

Arguments

`t`	vector of (real) numbers where the jacobian of the CF is evaluated; numeric.
`theta`	vector of parameters of the stable law; vector of length 4.
`pm`	parametrisation, an integer (0 or 1); default: `pm = 0` (Nolan's 'S0' parametrisation).

Details

The numerical derivation is obtained by a call to the function `jacobian` from package **numDeriv**. We have set up its arguments by default and the user is not given the option to modify them.

Value

a matrix `length(t)` $\times$ 4 of complex numbers.

See Also

`ComplexCF`

Examples

```
## define the parameters
nt <- 10
t <- seq(0.1, 3, length.out = nt)
theta <- c(1.5, 0.5, 1, 0)
pm <- 0

## Compute the jacobian of the characteristic function
jack_CF <- jacobianComplexCF(t = t, theta = theta, pm = pm)
```

KoutParametersEstim *Iterative Koutrouvelis regression method*

Description

Iterative Koutrouvelis regression method with different spacing schemes (points where the eCF is computed).

Usage

```
KoutParametersEstim(x, theta0 = NULL,
                    spacing = c("Kout", "UniformSpac", "ArithSpac", "free"),
                    pm = 0, tol = 0.05, NbIter = 10, PrintTime = FALSE, ...)
```

Arguments

`x`	data used to perform the estimation: vector of length n.
`theta0`	initial guess for the 4 parameters values: vector of length 4
`spacing`	scheme used to select the points where the moment conditions are evaluated. `Kout` is the scheme suggested by Koutrouvelis, `UniformSpac` and `ArithSpac` are the uniform and arithmetic spacing schemes over the informative interval $[\epsilon, A_n]$. If user choose free, he needs to provide a set of points `t_points` and `u_points` in `....`

pm	parametrisation, an integer (0 or 1); default: pm = 0 (Nolan's 'S0' parametrisation).
tol	the loop stops if the relative error between two consecutive estimation is smaller then tol; default = 0.05.
NbIter	maximum number of iteration. The loop stops when NbIter is reached; default = 10.
PrintTime	logical flag; if set to TRUE, the estimation duration is printed out to the screen in a readable format (h/min/sec).
...	other arguments to pass to the function. See Details.

Details

spacing

4 options for the spacing scheme are implemented as described above. In particular:

UniformSpac, ArithSpac: The user can specify the number of points to choose in both regression by inputting nb_t and nb_u. Otherwise the Koutrouvelis table will be used to compte them.

free: The user is expected to provide t_points and u_points otherwise the Kout scheme will be used.

Value

a list with the following elements:

Estim	list containing the vector of 4 parameters estimate (par), the 2 regressions objects (reg1 and reg2) and the matrix of iterations estimate (vals).
duration	estimation duration in a numerical format.
method	character describing the method used.

References

Koutrouvelis IA (1980). "Regression-type estimation of the parameters of stable laws." *Journal of the American Statistical Association*, **75**(372), pp. 918–928.

Koutrouvelis IA (1981). "An iterative procedure for the estimation of the parameters of stable laws: An iterative procedure for the estimation." *Communications in Statistics-Simulation and Computation*, **10**(1), pp. 17–28.

See Also

Estim

Examples

```
pm <- 0
theta <- c(1.45, 0.5, 1.1, 0.4)
set.seed(1235)
x <- rstable(200, theta[1], theta[2], theta[3], theta[4], pm = pm)
theta0 <- theta - 0.1
```

```
spacing <- "Kout"

KoutParametersEstim(x = x, theta0 = theta0,
                        spacing = spacing, pm = pm)
```

`McCullochParametersEstim` *Quantile-based method*

Description

McCulloch quantile-based method.

Usage

```
McCullochParametersEstim(x)
```

Arguments

`x`	data used to perform the estimation: vector of length n.

Details

`McCullochParametersEstim` is a wrapper for function `.qStableFit` from package **fBasics**.

Value

`numeric` of length 4, represening the values of the 4 parameters

References

McCulloch JH (1986). "Simple consistent estimators of stable distribution parameters." *Communications in Statistics-Simulation and Computation*, **15**(4), pp. 1109–1136.

See Also

`Estim`, `IGParametersEstim`

Examples

```
set.seed(333)
x <- rstable(500, 1.3, 0.4, 1, 0)
McCullochParametersEstim(x)
```

`MLParametersEstim` *Maximum likelihood (ML) method*

Description

Uses the numerical ML approach described by Nolan to estimate the 4 parameters of stable law. The method may be slow for large sample size due to the use of numerical optimisation routine.

Usage

```
MLParametersEstim(x, theta0 = NULL, pm = 0, PrintTime = FALSE, ...)
```

Arguments

`x`	data used to perform the estimation: vector of length n.
`theta0`	initial guess for the 4 parameters values: If `NULL`, the Kogon-McCulloch method is called, see `IGParametersEstim`; a vector of length 4.
`pm`	parametrisation, an integer (0 or 1); default: `pm=0` (Nolan's 'S0' parametrisation).
`PrintTime`	logical flag; if set to TRUE, the estimation duration is printed out to the screen in a readable format (h/min/sec).
`...`	Other argument to be passed to the optimisation function.

Details

The function performs the minimisation of the numerical (-)log-density of stable laws computed by function `dstable` from package **stabledist**.

After testing several optimisation routines, we have found out that the `"L-BFGS-B"` algorithm performs better with the ML method (faster, more accurate).

Value

a list with the following elements:

`Estim`	output of the optimisation function,
`duration`	estimation duration in a numerical format,
`method`	`character` describing the method used.

References

Nolan J (2001). "Maximum likelihood estimation and diagnostics for stable distributions." *L'evy processes: theory and applications*, pp. 379–400.

See Also

`Estim`

Examples

```
theta <- c(1.5, 0.4, 1, 0)
pm <- 0
## 50 points does not give accurate estimation
## but it makes estimation fast for installation purposes
## use at least 200 points to get decent results.
set.seed(1333)
x <- rstable(50, theta[1], theta[2], theta[3], theta[4], pm)

## This example takes > 30 sec hence commented out
## Not run:
  ML <- MLParametersEstim(x = x, pm = pm, PrintTime = TRUE)

## End(Not run)
## see the Examples folder for more examples.
```

`PrintDuration` *Print duration*

Description

Print duration in human readable format.

Usage

```
PrintDuration(t, CallingFct = "")
```

Arguments

`t`	Duration; `numeric` of `length` 1 or 3.
`CallingFct`	Name of the calling function.

Details

The duration will be printed in the format: hours/minutes/seconds.

Value

Prints a `character` to the screen.

Examples

```
ti <- getTime_()
for (i in 1:100) x = i*22.1
tf <- getTime_()
duration <- ComputeDuration(ti, tf)
PrintDuration(duration, "test")
```

PrintEstimatedRemainingTime *Estimated remaining time*

Description

Prints the estimated remaining time in a loop. Useful in Monte Carlo simulations.

Usage

```
PrintEstimatedRemainingTime(ActualIter, ActualIterStartTime, TotalIterNbr)
```

Arguments

`ActualIter`	Actual Iteration; `integer`
`ActualIterStartTime`	Actual Iteration Starting time; `numeric`
`TotalIterNbr`	Total number of iterations; `integer`

Details

Called at the end of each Monte Carlo step, this function will compute the duration of the actual step, an estimate of the remaining MC loops duration and prints the result to the screen in a human readable format using function `PrintDuration`.

See Also

`PrintDuration`, `ComputeDuration`.

RegularisedSol *Regularised Inverse*

Description

Regularised solution of the (ill-posed) problem $K\phi = r$ where K is a $n \times n$ matrix, r is a given vector of `length` n. Users can choose one of the 3 schemes described in Carrasco and Florens (2007).

Usage

```
RegularisedSol(Kn, alphaReg, r,
               regularization = c("Tikhonov", "LF", "cut-off"),
               ...)
```

Arguments

Kn	numeric $n \times n$ matrix.
alphaReg	regularisation parameter; numeric in]0,1].
r	numeric vector of length n.
regularization	regularization scheme to be used, one of "Tikhonov" (Tikhonov scheme), "LF" (Landweber-Fridmann) and "cut-off" (spectral cut-off). See Details.
...	the value of c used in the "LF" scheme. See Carrasco and Florens(2007).

Details

Following Carrasco and Florens(2007), the regularised solution of the problem $K\phi = r$ is given by :

$$\varphi_{\alpha_{reg}} = \sum_{j=1}^{n} q(\alpha_{reg}, \mu_j) \frac{< r, \psi_j >}{\mu_j} \phi_j,$$

where q is a (positive) real function with some regularity conditions and μ, ϕ, ψ the singular decomposition of the matrix K.

The regularization parameter defines the form of the function q. For example, the "Tikhonov" scheme defines $q(\alpha_{reg}, \mu) = \frac{\mu^2}{\alpha_{reg}+\mu^2}$.

When the matrix K is symmetric, the singular decomposition is replaced by a spectral decomposition.

Value

the regularised solution, a vector of length n.

References

Carrasco M, Florens J and Renault E (2007). "Linear inverse problems in structural econometrics estimation based on spectral decomposition and regularization." *Handbook of econometrics*, **6**, pp. 5633–5751.

See Also

solve

Examples

```
## Adapted from R examples for Solve
## We compare the result of the regularized sol to the expected solution

hilbert <- function(n) { i <- 1:n; 1 / outer(i - 1, i, "+")}

K_h8 <- hilbert(8);
r8 <- 1:8

alphaReg_robust <- 1e-4
Sa8_robust <- RegularisedSol(K_h8,alphaReg_robust,r8,"LF")
```

```
alphaReg_accurate <- 1e-10
Sa8_accurate <- RegularisedSol(K_h8,alphaReg_accurate,r8,"LF")

## when pre multiplied by K_h8, the expected solution is 1:8
## User can check the influence of the choice of alphaReg
```

`sampleComplexCFMoment` *Complex moment condition based on the characteristic function*

Description

Computes the moment condition based on the characteristic function as a complex vector.

Usage

```
sampleComplexCFMoment(x, t, theta, pm = 0)
```

Arguments

`x`	vector of data where the ecf is computed.
`t`	vector of (real) numbers where the CF is evaluated; numeric.
`theta`	vector of parameters of the stable law; vector of length 4.
`pm`	parametrisation, an integer (0 or 1); default: `pm=0` (Nolan's 'S0' parametrisation).

Details

The moment conditions

The moment conditions are given by:

$$g_t(X, \theta) = g(t, X; \theta) = e^{itX} - \phi_\theta(t)$$

If one has a sample $x_1, \ldots, x_n$ of i.i.d realisations of the same random variable X, then:

$$\hat{g}_n(t, \theta) = \frac{1}{n} \sum_{i=1}^{n} g(t, x_i; \theta) = \phi_n(t) - \phi_\theta(t),$$

where $\phi_n(t)$ is the eCF associated to the sample $x_1, \ldots, x_n$, and defined by $\phi_n(t) = \frac{1}{n} \sum_{j=1}^{n} e^{itX_j}$. The function compute the vector of difference between the eCF and the CF at a set of given point `t`.

Value

a complex vector of `length(t)`.

See Also

`ComplexCF`, `sampleRealCFMoment`

Examples

```
## define the parameters
nt <- 10
t <- seq(0.1, 3, length.out = nt)
theta <- c(1.5, 0.5, 1, 0)
pm <- 0

set.seed(222)
x <- rstable(200, theta[1], theta[2], theta[3], theta[4], pm)

## Compute the characteristic function
CFMC <- sampleComplexCFMoment(x = x, t = t, theta = theta, pm = pm)
CFMC
```

`sampleRealCFMoment` *Real moment condition based on the characteristic function*

Description

Computes the moment condition based on the characteristic function as a real vector.

Usage

```
sampleRealCFMoment(x, t, theta, pm = 0)
```

Arguments

`x`	vector of data where the ecf is computed.
`t`	vector of (real) numbers where the CF is evaluated; numeric.
`theta`	vector of parameters of the stable law; vector of length 4.
`pm`	Parametrisation, an integer (0 or 1); default: `pm=0` (Nolan's 'S0' parametrisation).

Details

The moment conditions

The moment conditions are given by:

$$g_t(X, \theta) = g(t, X; \theta) = e^{itX} - \phi_\theta(t).$$

If one has a sample $x_1, \ldots, x_n$ of i.i.d realisations of the same random variable X, then:

$$\hat{g}_n(t, \theta) = \frac{1}{n} \sum_{i=1}^{n} g(t, x_i; \theta) = \phi_n(t) - \phi_\theta(t),$$

where $\phi_n(t)$ is the eCF associated with the sample $x_1, \ldots, x_n$, and defined by $\phi_n(t) = \frac{1}{n} \sum_{j=1}^{n} e^{itX_j}$.

The function compute the vector of difference between the eCF and the CF at a set of given point `t`. If `length(t)` = `n`, the resulting vector will be of `length` = `2n`, where the first `n` components will be the real part and the remaining the imaginary part.

Value

a vector of length `2 * length(t)`.

See Also

`ComplexCF`, `sampleComplexCFMoment`

Examples

```
## define the parameters
nt <- 10
t <- seq(0.1, 3, length.out = nt)
theta <- c(1.5, 0.5, 1, 0)
pm <- 0

set.seed(222)
x <- rstable(200, theta[1], theta[2], theta[3], theta[4], pm)

# Compute the characteristic function
CFMR <- sampleRealCFMoment(x = x, t = t, theta = theta, pm = pm)
CFMR
```

`StatFcts` *Default functions used to produce the statistical summary*

Description

Default functions used to produce the statistical summary in the Monte Carlo simulations.

Details

The functions are:

Mean `.mean <- function(p,...) mean(p)`

Min `.min <- function(p,...) min(p)`

Max `.max <- function(p,...) max(p)`

Sn `.Sn <- function(p,n,...) sqrt(n)*sd(p)`

MSE `.MSE <- function(p,paramT,...) (1/length(p))*sum((p-paramT)^2)`

Std error `.st.err <- function(p,...) sd(p)/sqrt(length(p))`

To change the statistical summary, provide functions with similar signatures and pass a character vector containing the function names to `Estim_Simulation`.

TexSummary *LaTeX summary*

Description

Creates a TeX table from a summary object or a vector of files.

Usage

```
TexSummary(obj, files = NULL, sep_ = ",", FctsToApply = StatFcts,
           caption = "Statistical Summary", label = "Simtab",
           digits = 3, par_index = 1, MCparam = 1000, ...)
```

Arguments

obj	list of length 4 containing a summary matrix object associated to each parameter identical to the one produced by function ComputeStatObjectFromFiles.
files	character vector containing the files name to be parsed. It will be passed to function ComputeStatObjectFromFiles.
sep_	field separator character to be passed to function ComputeStatObjectFromFiles.
FctsToApply	functions used to produce the statistical summary to be passed to the function ComputeStatObjectFromFiles.
caption	character vector with length equal to length(par_index) containing the table's caption or title.
label	character vector with length equal to length(par_index) containing the LaTeX label.
digits	numeric vector of length equal to one (in which case it will be replicated as necessary) or to the number of columns of the resulting table or length of FctsToApply or matrix of the same size as the resulting table indicating the number of digits to display in the corresponding columns. See xtable.
par_index	numeric or character vector of length 1, 2, 3 or 4 of the desired indices to be selected in obj. See Details.
MCparam	number of Monte Carlo simulations for each couple of parameters, default = 1000; integer.
...	other arguments to be passed to function ComputeStatObjectFromFiles.

Details

Accepted values for par_index are c(1,2,3,4) or c("alpha","beta","gamma","delta") or mixed.

Some examples are provided in the example folder.

Value

a `list` of length `length(par_index)` whose elements are objects from class `Latex` (produced by `toLatex`)

See Also

`Estim_Simulation`, `ComputeStatObjectFromFiles`, `xtable`

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